MAT 2377A
Final Examination

April 2004 **Professor G. Ivanoff**
Time: 3 hours

Student Number:____________

Family Name: ______________ **First Name:** ______________

- **This is an open book examination. Calculators are permitted.**
- **Record your answer to each question in the table below. Each question is worth 4 marks.**
- **At the end of the examination, hand in only this page.**

Question	Answer	Question	Answer
1		14	
2		15	
3		16	
4		17	
5		18	
6		19	
7		20	
8		21	
9		22	
10		23	
11		24	
12		25	
13			

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Professor's use only: Grade=______/25

1. Suppose that 10% of the radios produced by a certain factory are defective. An engineer tests a random sample of 250 radios. Approximate the probability that there are at most 20 defective radios in the sample.

   a) 0.7996 b) 0.5 c) 0.1469 d) 0.019 e) 0.2005

2. Consider the situation described in problem 1. On average, how many radios would be tested in order to find the first defective?

   a) 5 b) 15 c) 250 d) 90 e) 10

3. Let $X_1, ..., X_{10}$ be a random sample from a population with mean μ. If $\mu \neq 0$, which of the following estimators of μ are unbiased?

$$\begin{aligned}
\hat{\mu}_1 &= 2X_2 + 2\,X_2 + 2\,X_4 + 2\,X_6 + 2\,X_8 - 9\,X_{10} \\
\hat{\mu}_2 &= \alpha\,X_5 + (1-\alpha)\,X_6, \text{ where } \alpha \text{ is an arbitrary constant} \\
\hat{\mu}_3 &= \frac{1}{10}\sum_{i=1}^{10} X_i \\
\hat{\mu}_4 &= \frac{2\,X_1}{3} - \frac{X_2 + X_3}{3}
\end{aligned}$$

   a) $\hat{\mu}_3$ only b) $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\mu}_3$ and $\hat{\mu}_4$ c) $\hat{\mu}_1$, $\hat{\mu}_2$ and $\hat{\mu}_3$ d) $\hat{\mu}_4$ only
   e) $\hat{\mu}_2$ and $\hat{\mu}_3$

4. Let $X_1, ..., X_{10}$ be a random sample from a population with mean μ and variance 2. Consider the following estimators for μ:

$$\hat{\mu}_5 = \frac{X_5}{5} + \frac{4\,X_6}{5} \qquad \text{and} \qquad \hat{\mu}_6 = \frac{3\,X_1}{4} + \frac{X_2}{4}.$$

   Find the variance of each of the estimators.

   a) $Var(\hat{\mu}_5) = 17/25$, $Var(\hat{\mu}_6) = 5/8$
   b) $Var(\hat{\mu}_5) = 1$, $Var(\hat{\mu}_6) = 1$
   c) $Var(\hat{\mu}_5) = 34/25$, $Var(\hat{\mu}_6) = 5/4$
   d) $Var(\hat{\mu}_5) = 68/25$, $Var(\hat{\mu}_6) = 5/2$
   e) none of the preceding

5. Suppose that the length of a plastic strip is normally distributed with mean 5 cm. and standard deviation .5 cm. What proportion of plastic strips are *not* between 4.5 and 5.5 cm. in length?

   a) 0.6827 b) 0.5 c) 0.1587 d) 0.8413 e) 0.3173

6. A widget manufacturer has two factories, A and B. 30% of the widgets are made in factory A, and the remainder in B. Suppose that 95% of the widgets produced by factory A meet specifications while only 85% of the widgets produced by factory B meet specifications. If I buy a widget made by this manufacturer, what is the probability that it meets specifications?

   a) insufficient information b) 0.90 c) 0.95 d) 0.85 e) 0.88

7. Consider the situation described in problem 6. If the widget I buy meets specifications, what is the probability that it was made in factory A?

   a) insufficient information b) 0.3239 c) 0.5 d) 0.4325 e) 0.7325

8. Suppose that the weight of a female fitness instructor has a mean of 55 kg. and a standard deviation of 5 kg. A group of 36 female fitness instructors is travelling from Sudbury to Ottawa to attend a seminar. What is the probability that the average weight of the women in the group is more than 57 kg.?

   a) 0.0082 b) 0.9918 c) approximately 0 ($< 10^{-5}$) d) 0.6554 e) 0.3446

9. Let X be a discrete random variable with the following p.m.f,:

x	-1	0	1	2
$f(x)$	0.2	0.1	0.2	0.5

Find the mean $\mu = E[X]$ and the variance $\sigma^2 = Var(X)$ of X.

a) $\mu = 1.0$, $\sigma^2 = 1.4$ b) $\mu = 1.0$, $\sigma^2 = 2.4$
c) $\mu = 1.5$, $\sigma^2 = 0.15$ d) $\mu = 0.5$, $\sigma^2 = 2.15$
e) $\mu = 1.5$, $\sigma^2 = 1.0$

10. Let X and Y be independent random variables such that X is Poisson with mean 2 and Y is binomial with $n = 5$ and $p = 0.2$. Find

$$P(X = 0 \text{ or } Y = 0) = P(\{X = 0\} \cup \{Y = 0\}).$$

a) insufficient information b) 0.4630 c) 0.4187 d) 0.4325 e) 0.5621

11. Suppose that events A_1, A_2, A_3 and A_4 are mutually exclusive and exhaustive. If $P(A_1) = P(A_2) = 0.25$ and $P(A_3) = 0.2$, find

$$P(A_2 \cup A_4).$$

a) 0.65 b) 0.8 c) 0.75 d) 0.55 e) insufficient information

12. Suppose that the requests for service at an information centre follow a Poisson process with an average of 3 requests per day. What is the probability that there will be exactly 5 requests in 4 days?

a) 0.0347 b) 0.2573 c) 0.0677 d) 0.1008 e) 0.0127

13. The joint probability mass function of discrete random variables X and Y is given below:

x \ y	1	2	3	4
1	1/30	2/30	4/30	3/30
2	2/30	4/30	5/30	9/30

Find $P(X = 2, Y \geq 3))$.

a) 0 b) 5/30 c) 14/30 d) 6/30 e) 4/30

14. Using the p.m.f. from problem 13, find the covariance σ_{XY}.

(a) 0 (b) -1/30 (c) 151/30 (d) 1/30 (e) -151/30

15. Using the p.m.f. from problem 13, find the conditional probability that $Y = 3$ if it is known that $X = 1$ (i.e., find $P(\{Y = 3\}|\{X = 1\})$).

(a) 4/30 (b) 9/30 (c) 4/5 (d) 4/10 (e) 9/10

16. Let $\overline{X}$ and S^2 be the sample mean and sample variance of a random sample of size 10 from a $N(4, 9)$ distribution. Find a constant c such that

$$P\left(\frac{\overline{X} - 4}{S/\sqrt{10}} \leq c\right) = .95$$

(a) 1.833 (b) 1.86 (c) 1.645 (d) 2.262 (e) 1.812

17. Suppose that in problem 16, the true mean μ is unknown. If we want to be at least 95% certain that $|\overline{X} - \mu| \leq 1$, what is the smallest sample size that we should take?

(a) 34 (b) 25 (c) 35 (d) 24 (e) 10

18. The time taken in minutes to complete a particular task in a widget factory follows a normal distribution with mean 30 and variance 1. In an effort to reduce the mean time required for the task, 16 workers are asked to carry out the task using a new procedure. The procedure will be adopted (i.e. $H_0 : \mu = 30$ will be rejected and $H_1 : \mu < 30$ will be accepted) if the 16 workers take an average of less than 29.4 minutes to complete the task. Assuming that the variance remains unchanged when the new procedure is used, calculate
(i) α, the probability of a type I error, and
(ii) $\beta(29)$, the probability of a type II error if in fact $\mu = 29$.

(a) $\begin{array}{c}\alpha = .2743 \\ \beta = .3446\end{array}$ (b) $\begin{array}{c}\alpha \approx 0 \\ \beta \approx 0\end{array}$ (c) $\begin{array}{c}\alpha = .4404 \\ \beta = .4602\end{array}$ (d) $\begin{array}{c}\alpha = .0082 \\ \beta = .0548\end{array}$
(e) insufficient information

19. Consider the situation described in problem 18, but suppose now that we cannot assume that the variance remains unchanged when the new procedure is used. The times (in minutes) taken by the 16 workers yield the following data: $\overline{x} = 29.5, s = 1.2$. For a test of $H_0 : \mu = 30$ versus $H_1 : \mu < 30$, find bounds for the p-value of the appropriate test statistic, and state your conclusion if $\alpha = .05$.

(a) $.05 < p < .10$, do not reject H_0 (b) $.05 < p < .10$, reject H_0
(c) $.025 < p < .05$, do not reject H_0 (d) $.025 < p < .05$, reject H_0
(e) none of the preceding

20. An engineer wants to test $H_0 : \mu = 10$ versus $H_1 : \mu \neq 10$ at level $\alpha = .05$, where μ is the mean weight of a steel ingot (in kilograms). If the weight follows a normal distribution with a variance of 1, what sample size should be taken in order that $\beta(11) = .1$? ($\beta(11)$ is the probability of a type II error if $\mu = 11$.)

(a) 8 (b)10 (c) 14 (d) 9 (e) none of the preceding

21. Let Y denote the grade that a student will obtain in MAT 2377 and x the student's grade in first year calculus. Find the estimated least squares regression line of Y on x if a sample of 10 students yielded the following data:
$\sum_{i=1}^{10} x_i = 810$, $\sum_{i=1}^{10} y_i = 740$, $\sum_{i=1}^{10} x_i^2 = 66222$, $\sum_{i=1}^{10} y_i^2 = 55366$, $\sum_{i=1}^{10} x_i y_i = 60534$

(a) $\hat{y} = -.04 + .91x$ (b) $\hat{y} = -4.6 + .97x$ (c) $\hat{y} = 13.66 + .91x$
(d) $\hat{y} = 9.22 + .97x$ (e) $\hat{y} = -7.29 + 1.09x$

22. Using the data from question 21, find an estimate of the variance of Y.

(a) 29.82 (b) 2.98 (c) 3.73 (d) 3.31 (e) none of the preceding

23. A production process produces wire filaments. In order to control the mean tensile strength of the filaments, samples of size 4 are taken. Use the data below from 10 preliminary samples to determine upper and lower trial control limits for an $\overline{X}$ chart:

$$\sum_{i=1}^{10} \overline{x}_i = 28.6, \quad \sum_{i=1}^{10} r_i = 12.3$$

(a) UCL = 37.57, LCL = 19.63 (b) UCL = 3.57, LCL = 2.15 (c) UCL = 3.76, LCL = 1.96
(d) UCL = 3.24, LCL = 2.48 (e) UCL = 32.4, LCL = 24.8

24. Using the data in problem 23, estimate the standard deviation of the tensile strength of a filament.

(a) $\hat{\sigma} = 5.97$ (b) $\hat{\sigma} = 5.29$ (c) $\hat{\sigma} = 1.31$ (d) $\hat{\sigma} = 0.60$ (e) $\hat{\sigma} = 0.53$

25. An integrated-circuit chip has a reliability of .9 . Find the reliability of a component consisting of 3 such chips connected

   (a) in series

   (b) in parallel.

(a)	(a) .999 (b) .729	(b)	(a) .729 (b) .999	(c)	(a) .7 (b) .993	(d)	(a) .9 (b) .1

(e) none of the preceding